

# Theoretical comparison between two different filtering techniques suitable for the VLSI spectroscopic amplifier ROTOR

C.Fiorini<sup>a</sup>,M.Porro<sup>a\*</sup>,L.Strüder<sup>b</sup>

<sup>a</sup>Politecnico di Milano, Dipartimento di Elettronica e Informazione, Milano, Italy

<sup>b</sup>MPI für Extraterrestrische Physik Halbleiterlabor, München, Germany

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## Abstract

The development of new detection systems based on arrays of Silicon Drift Detectors (SDD) used for new X-ray spectroscopy applications, like X-ray Holography and EXAFS experiments, requires the realization of suitable integrated low-noise electronics for the readout of the detector signals. Recently a new VLSI time variant signal processor called ROTOR has been developed. Despite its time variant nature ROTOR is capable of correctly processing events randomly distributed along the time axis thanks to the employment of the Concurrent Wheel Technique (CWT). Two different possible solutions for the ROTOR chip have been developed, both suitable for the CWT working mechanism. A theoretical comparison between the noise performances of the two filtering methods has been carried out and is presented in this work.

*Keywords:* Type your keywords here, separated by semicolons ;

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## 1. Introduction

In recent years, the availability of intense photon flux provided by new high brilliance sources, like synchrotron light source, has brought to the development of new X-Ray spectroscopy applications, like X-Ray Holography [1] and EXAFS experiments [2].

Such applications require multi-element detectors with high-rate and high-resolution capabilities. In particular, detection systems based on arrays of

Silicon Drift Detectors (SDD) with on-chip electronics [3] have been developed.

These detectors allow to operate with good energy resolution at high counting rates near room temperature simply by means of a Peltier cooler [4].

The number of channels (about 1000) foreseen to fully exploit the intense photon flux generated by a synchrotron source required the realization of a suitable low noise integrated readout electronics, in order to limit the size and the power consumption of the system.

The main requirements for this electronics are:

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\* Corresponding author. Tel.: +39-02-2399-3744; fax: +39-02-2367-604; e-mail: porro@elet.polimi.it.

a) capability of handling asynchronous events with no trigger signal

b) energy resolution below 300eV @ 6 KeV, using a SDD

c) compatibility with monolithic integration in CMOS technology

d) maximum power consumption of 30mW/channel

Recently a new VLSI time variant signal processor, called ROTOR, has been realized [5,6].

Despite its time variant nature, ROTOR is capable of correctly processing events randomly distributed along the time axis even if no trigger signal is available, thanks to the employment of the Concurrent Wheel Technique (CWT).

This time variant approach makes it possible to obtain a finite-width weighting function, keeping all the advantages of the classical continuous time spectroscopy amplifiers (no trigger required, very low level discrimination).

Two different possible implementations of the ROTOR amplifier have been developed, both suitable for the CWT working mechanism.

The first implements a trapezoidal weighting function by means of the Switched Current Technique (SCT). The second performs a quasi trapezoidal weighting function, obtained by the sum of eight exponential functions, by means of the Multi Correlated Double Sampling (MCDS).

In this work we present a theoretical comparison between the noise performances of the two systems. The noise coefficients used to evaluate the Equivalent Noise Charge (ENC) of the two filters have been calculated and are discussed in this paper. Moreover, additional causes of loss of resolution due to the use of the CWT have been investigated, taking also into account the circuitual complexity for the realization of the two different solutions.

## 2. Working Principle of the Concurrent Wheel Technique

As can be better understood with the help of Fig.1, the CWT working principle is based on the use of a certain number (4 in Fig.1) of processors, called wheels, working in parallel; each one performs a flat-topped weighting function cyclically.

The weighting functions are suitably shifted one with respect to the other, in order that the sequence of the flat-tops covers the whole time axis. In this way, for any arrival time of the signal, there is at least one wheel that processes it with the maximum gain, i.e. with its flat-top.

The outputs of the different wheels are sequentially sampled and held on a common capacitor at the end of the corresponding weighting functions. The voltage on the hold capacitor during the sequence of the different measurements results in a "staircase-like" waveform. The peak value of this staircase corresponds to the wheel that has processed the signal with its flat-top. It can be stretched and converted by an ADC.

As already mentioned, each wheel must perform a flat-topped weighting function cyclically.  $T_{CYCLE}$  is defined as the time interval between the beginning of a weighting function and the beginning of the next one belonging to the same wheel. It is greater than the time length of the weighting function  $T_{WIDTH}$ , because some time is needed to acquire the signal and reset the circuit. It is now clear from Fig.2 that the minimum number of wheels

$$N = \frac{T_{CYCLE}}{T_{FLAT-TOP}} \quad \text{needed to cover the time axis is:}$$

where  $T_{FLAT-TOP}$  is the time length of the flat-top.

$N$  decreases as the ratio between the flat-top and the width of the weighting function increases.

However, increasing the length of the flat-top, the noise performances of the filter are worsened, as it will be shown in the following paragraphs. So a compromise between energy resolution and number of wheels to be implemented in a practical realization has to be achieved.

Eventually it can be stated that the total width of the weighting function is to be chosen shorter than the inverse of the counting rate:

$$T_{\text{Average}} \gg T_{\text{WIDTH}}$$

where  $T_{\text{Average}}$  is the average time between two input signals.

### 3. Switched Current Technique

The SCT [5] allows to realize a trapezoidal weighting function as shown in Fig. 3. How to obtain such a weighting function can be better explained with the help of Fig. 4. The output of the preamplifier is converted into the two currents  $I_+$  and  $I_-$ , having the same amplitude, proportional to the input pulse, and opposite signs. The timing of the circuit, controlled by a digital shift register integrated on the chip, consists of four intervals  $T_{\text{INT1}}$ ,  $T_{\text{FT}}$ ,  $T_{\text{INT2}}$  and  $T_{\text{SERV}}$ . During  $T_{\text{INT1}}$  the current  $I_+$  is integrated, during  $T_{\text{FT}}$  no current is integrated and during  $T_{\text{INT2}}$  current  $I_-$  is integrated.  $T_{\text{SERV}}$  is needed to acquire the output signal  $V_{\text{OUT}}$  at the end of the second integration and to reset the circuit.  $V_{\text{OUT}}$  is equal to:

$$V_{\text{OUT}} = \int_0^{T_{\text{INT1}}} I(t) - \int_{T_{\text{INT1}}+T_{\text{FT}}}^{T_{\text{INT1}}+T_{\text{FT}}+T_{\text{INT2}}} I(t)$$

where  $I(t) = I_+(t) = -I_-(t)$ .

Fig. 4 shows the output values at the measurement time  $T_M$  for different arrival times of the input signal.

It is clear that if the input signal arrives between the two integrations (during  $T_{\text{FT}}$ ), it is amplified with the maximum gain.

If we assume to have a  $\delta$ -like input pulse, the obtained weighting function gets closer to the trapezoidal shape as the output of the preamplifier approaches the sharp step shape, i.e. as the bandwidth of the preamplifier can be approximated to be infinity.

### 4. Multi Correlated Double Sampling

Fig. 5 shows the working principle of the MCDS. In this case the signal, instead of being integrated, is sampled eight times.

Again the cyclical timing of the circuit can be divided into the four parts  $T_{\text{S1}}$ ,  $T_{\text{FT}}$ ,  $T_{\text{S2}}$  and  $T_{\text{SERV}}$ . During  $T_{\text{S1}}$  the output of the preamplifier is sampled 4 times. During  $T_{\text{FT}}$  no samples are taken and during  $T_{\text{S2}}$  a series of 4 more samples is taken.

The samples belonging to the two different series have opposite sign. In this way, at the measurement time  $T_M$ , following the last sample, the output voltage of the filter is:

$$V_{\text{OUT}} = \sum_{i=1}^4 V(i) - \sum_{i=5}^8 V(i)$$

where  $V(i)$  is the voltage at the output of the preamplifier at sampling instant  $i$ .

Again it is evident that if the input signal arrives between the two series of samplings it is amplified with the maximum gain.

As the output of the preamplifier approaches the sharp step shape, the weighting function gets closer to the staircase shown in Fig. 6.

As it will be clear later, this is not acceptable, because the series noise, depending on the derivative of the weighting function, would increase indefinitely. It follows that some bandwidth limitation of the preamplifier must be introduced to obtain the smoothed weighting function also shown in Fig. 6 and reduce the series noise. This is not necessary in the SCT case.

It is now worthwhile to point out that the limited-bandwidth weighting function of Fig. 6 doesn't have a real flat-region. As it will be evaluated in detail later, this will cause an additional loss of resolution when processing asynchronous signals with the CWT. It will be so necessary to find a compromise between series noise and bandwidth of the circuit.

## 5. ENC and noise coefficients

To make a comparison between the two different filtering techniques, it is first of all necessary to evaluate the coefficients  $A_1$ ,  $A_2$  and  $A_3$  of the well known expression [7] of the Equivalent Noise Charge (ENC):

$$ENC^2 = \frac{a}{\tau} C_T^2 A_1 + \left[ 2\pi a_f C_T^2 + \frac{b_f}{2\pi} \right] A_2 + b\tau A_3$$

$A_1$  weights the white series noise,  $A_2$  the 1/f noise and  $A_3$  the white parallel noise. They can be calculated using the following time domain expressions:

$$A_1 = \int_{-\infty}^{+\infty} [w'(y)]^2 dy \quad (1.a)$$

$$A_2 = \int_{-\infty}^{+\infty} [w^{(1/2)}(t)]^2 dt \quad (1.b)$$

$$A_3 = \int_{-\infty}^{+\infty} [w(y)]^2 dy \quad (1.c)$$

where  $w(t)$  is the weighting function and  $y=t/\tau$  is the time normalized to the shaping time  $\tau$ .  $w^{(1/2)}$  is the derivative of order  $1/2$  of the weighting function [8].

From (1.a) it is clear that the series noise increases indefinitely as the slope of the weighting function increases.

### 5.1. SCT Noise Coefficients

Fig. 7a shows the definitions used in the calculation of the noise coefficients for the SCT weighting function  $w_{SCT}(t)$ .

Defining the shaping time  $\tau$  as the time interval corresponding to the first integration,  $(\alpha-1)\tau$  as the flat-top length and  $(\alpha+1)\tau$  as the total width of the weighting function,  $w_{SCT}(t)$  can be written as:

$$w_{SCT}(t) = \frac{1}{\tau} [tu(t) - (t-\tau)u(t-\tau) - (t-\alpha\tau)u(t-\alpha\tau) + (t-(\alpha+1)\tau)u(t-(\alpha+1)\tau)]$$

where  $u(t)$  is the Heaviside function. The function  $w_{SCT}(t)$  is assumed to be normalized to unity peak value.  $\text{Max}[w_{SCT}(t)]=1$ .

With these definitions the noise coefficients can be calculated with (1) as:

$$A_1 = 2$$

$$A_2 = \frac{1}{\pi} [(\alpha+1)^2 \ln(\alpha+1) + (\alpha-1)^2 \ln(\alpha-1) - 2\alpha^2 \ln \alpha]$$

$$A_3 = \alpha - \frac{1}{3}$$

Fig. 8 and 9 show  $A_1, A_2$  and  $A_3$  as functions of  $\alpha$ , i.e. as functions of the flat-top length. As we can expect  $A_1$  is constant, since the slope of the flat-top is zero, while  $A_2$  and  $A_3$  increase logarithmically and linearly respectively.

We can conclude that the 1/f noise and the white parallel noise increase as the flat-top length increases.

### 5.2. MCDS Noise Coefficients

We have calculated the SCT noise coefficients without taking into account the effects of the finite bandwidth of the circuit. In other words, we have supposed to have a sharp step waveform at the output of the preamplifier, as the response to a  $\delta$ -like input pulse. In practice it is possible to choose a preamplifier sufficiently fast to make the effect of its finite bandwidth negligible. This is not possible in the MCDS case, since a bandwidth limitation must be introduced to limit the series noise. It follows that a parameter describing the bandwidth of the circuit should appear in the expression of the weighting function.

Under these consideration the MCDS folded weighting function  $w_{MCDS}(t)$  can be written as:

$$\begin{aligned}
w_{MCDS}(t) = & \frac{1}{4} \left[ (1 - e^{-\frac{t}{q}})u(t) + (1 - e^{-\frac{1}{q}(t-\frac{1}{4}\tau)})u(t - \frac{1}{4}\tau) \right. \\
& + (1 - e^{-\frac{1}{q}(t-\frac{1}{2}\tau)})u(t - \frac{1}{2}\tau) + (1 - e^{-\frac{1}{q}(t-\frac{3}{4}\tau)})u(t - \frac{3}{4}\tau) \\
& - (1 - e^{-\frac{t-\alpha\tau}{q}})u(t - \alpha\tau) - (1 - e^{-\frac{1}{q}(t-(\alpha+\frac{1}{4})\tau)})u(t - (\alpha + \frac{1}{4})\tau) \\
& - (1 - e^{-\frac{1}{q}(t-(\alpha+\frac{1}{2})\tau)})u(t - (\alpha + \frac{1}{2})\tau) \\
& \left. - (1 - e^{-\frac{1}{q}(t-(\alpha+\frac{3}{4})\tau)})u(t - (\alpha + \frac{3}{4})\tau) \right]
\end{aligned}$$

$w_{MCDS}(t)$ <sup>1</sup> is normalized in such a way that:

$$\lim_{q \rightarrow 0} \text{Max}[w_{MCDS}(t)] = 1$$

As shown in Fig. 7b the shaping time  $\tau$  is defined as four times the time interval between two samplings of the same series.  $q$  is the time constant of the exponential functions (depending on the bandwidth of the preamplifier),  $(\alpha-1)\tau$  is the nominal flat-top and  $(\alpha+1)\tau$  is the nominal width of the weighting function.

Putting  $k=q/\tau$ ,  $A_1$  and  $A_3$  can be determined as:

$$\begin{aligned}
A_1 = & \frac{1}{16k} \left[ 4 + 6e^{-\frac{1}{4k}} - e^{-\frac{4\alpha-3}{4k}} + 4e^{-\frac{1}{2k}} - 2e^{-\frac{2\alpha-1}{2k}} \right. \\
& + 2e^{-\frac{3}{4k}} - 3e^{-\frac{4\alpha-1}{4k}} - 4e^{-\frac{\alpha}{k}} - 3e^{-\frac{1+4\alpha}{4k}} - 2e^{-\frac{1+2\alpha}{2k}} \\
& \left. - e^{-\frac{3+4\alpha}{4k}} \right]
\end{aligned}$$

$$\begin{aligned}
A_3 = & \alpha - \frac{5}{16} + \frac{1}{4}k \left[ \frac{3}{4}e^{-\frac{1+4\alpha}{4k}} - 1 - e^{-\frac{1}{2k}} - \frac{3}{2}e^{-\frac{1}{4k}} \right. \\
& + \frac{1}{4}e^{-\frac{3+4\alpha}{4k}} + \frac{1}{2}e^{-\frac{1+2\alpha}{2k}} - \frac{1}{2}e^{-\frac{3}{4k}} + e^{-\frac{\alpha}{k}} \\
& \left. + \frac{1}{4}e^{-\frac{4\alpha-3}{4k}} + \frac{1}{2}e^{-\frac{2\alpha-1}{2k}} + \frac{3}{4}e^{-\frac{4\alpha-1}{4k}} \right]
\end{aligned}$$

<sup>1</sup> The use of the folded expression is justified by simpler calculations and it doesn't affect in any way the obtained results.

With regard to the coefficient  $A_2$ , it has been necessary to make a numerical integration with the PC. Table 1 reports the obtained values. It is possible to see, also from Fig. 10 and 11, that  $A_1$  is almost independent on  $\alpha$  and  $A_2$  and  $A_3$  increase with  $\alpha$  approximately with the same trend as in the SCT case.  $A_1, A_2$  and  $A_3$  are then decreasing functions of  $k$ . What is really important to notice is that  $A_1$ , and therefore the series noise, increases indefinitely as  $k$  decreases, i.e. as the bandwidth of the circuit increases. On the other hand it must be considered that in the MCDS case a real flat region doesn't exist, since also the nominal flat-top has a non-zero slope. Limiting the bandwidth of the circuit the series noise is reduced, but the slope of the flat-top increases.

## 6. Effects of the non-zero slope of the flat-top in MCDS case

Since the input signals have a random occurrence in time, they can hit any point of the flat-top and a big slope can result in significantly different output amplitudes related to the same input pulse.

The flat-top expression is:

$$\begin{aligned}
w_{flat,MCDS}(t) = & \frac{1}{4} \left[ 4 - e^{-\frac{t}{k\tau}} - e^{-\frac{1}{k\tau}(t-\frac{1}{4}\tau)} \right. \\
& \left. - e^{-\frac{1}{k\tau}(t-\frac{1}{2}\tau)} - e^{-\frac{1}{k\tau}(t-\frac{3}{4}\tau)} \right]
\end{aligned} \quad (2)$$

where  $\tau \leq t \leq \alpha\tau$ .

Considering the input events uniformly distributed in time, i.e. considering the arrival time of the signal a random variable  $T$  with distribution

$$f_T(t) = \frac{1}{(\alpha-1)\tau}$$

in the interval  $\tau \leq t \leq \alpha\tau$ , the probability density of the flat-top values  $w_{flat}$  that process the signal can be calculated from (2) with the transformation method [9]. It turns out to be:

$$f_{W_{flat}}(w_{flat}) = \frac{k}{(\alpha-1)(1-w_{flat})}$$

with  $w_{\text{MCDS}}(\tau) \leq w_{\text{flat}} \leq w_{\text{MCDS}}(\alpha\tau)$

It is clear that the standard deviation  $\sigma_{w,\text{flat}}$  of this distribution decreases as  $\alpha$  increases, but –as seen in the previous paragraph– the noise increase as the flat-top gets wider. Moreover  $\sigma_{w,\text{flat}}$  goes to zero with  $k$ , but the series noise increases indefinitely.

The signal to noise ratio can be expressed in terms of r.m.s. values as:

$$\frac{S}{N} = \frac{Q_{\text{IN}}}{\sqrt{\text{ENC}^2 + Q_{\text{IN}}^2 \frac{\sigma_{w,\text{flat}}^2}{w_{\text{flat}}^2}}}$$

It is now important to point out that the noise term at the denominator depends also on the input charge  $Q_{\text{IN}}$ . The shape of the spectrum of the signal at the multichannel analyzer is not gaussian anymore and depends on the input charge. The spectrum is now the convolution of the gaussian waveform due to the electronic noise with the distribution of the output values  $\rho_{\text{out}}^2$ :

$$f_{Q_{\text{out}}}(\rho_{\text{out}}) = \frac{k}{(\alpha-1)(Q_{\text{IN}} - \rho_{\text{out}})}$$

where  $Q_{\text{IN}} w_{\text{MCDS}}(\tau) \leq \rho_{\text{out}} \leq Q_{\text{IN}} w_{\text{MCDS}}(\alpha\tau)$ , due to the slope of the nominal flat-top.

We should have such a  $\sigma_{w,\text{flat}}$  that makes the term

$$Q_{\text{IN}} \frac{\sigma_{w,\text{flat}}}{w_{\text{flat}}}$$

small with respect to the ENC term for any possible charge in the dynamic input range.

Given  $k$  and  $\alpha$ , to estimate the maximum error, we evaluate the line broadening related to the maximum input signal. For example with  $k=0.076$ ,  $\alpha=2$  and an input charge of 8333 electrons (corresponding to 30 KeV using a SDD), it is:

$$Q_{\text{IN}} \frac{\sigma_{w,\text{flat}}}{w_{\text{flat}}} = 15 \text{ electrons r.m.s.}$$

<sup>2</sup> Output values have here the dimension of a charge, since the weighting function is normalized.

This is a relatively high contribution that may not be tolerated in high-resolution X-ray spectroscopy with SDDs.

Since, as already stated, we cannot vary the parameters  $\alpha$  and  $k$  beyond certain limits because of the electronic noise, we must operate in a different way, in order to reduce the loss of resolution associated to the non-zero slope of the flat-top. Given suitable values of  $\alpha$  and  $k$ , we define an effective flat-top (Fig. 7b), shorter than the nominal one, in which the change of the gain between the minimum and the maximum value is less than a given quantity  $\varepsilon \text{Max}[w_{\text{MCDS}}(t)]$  (with e.g.  $\varepsilon=1\%$ ). We must now cover the time axis with the sequence of the effective flat-tops, instead of the nominal ones. Obviously in this way more wheels may be needed.

## 7. Comparison between SCT and MCDS using the CWT

A possible comparison between SCT and MCDS can be made using the systems with the same counting rate, i.e. equating the total width of the two weighting functions. Fig. 7 shows the parameters used to make the comparison. Given  $\alpha$ ,  $\varepsilon$  and  $k$  the width of the effective flat-top in MCDS is:

$$FT_{\text{MCDS,Eff}} = (\alpha-1)\tau(1-\delta)$$

where  $\delta = \delta(\alpha, \varepsilon, k)$ .

Table 2 reports the values of  $\delta$  for different values of  $\alpha$  and  $k$  with  $\varepsilon=1\%$ .

The time interval between the beginning of the nominal flat-top and the effective one is  $(\alpha-1)\tau\delta$ . In the same way the weighting function takes a time equal to  $(\alpha-1)\tau\delta$  from the point  $(\alpha+1)\tau$  (nominal width) to reach an amplitude value whose difference from zero is approximately  $\varepsilon \text{Max}[w_{\text{MCDS}}(t)]$ . So we can define the effective width of the weighting function for MCDS as:

$$W_{\text{MCDS,Eff}} = \tau[(\alpha+1) + (\alpha-1)\delta]$$

The ratios between the effective flat-top (equal to the nominal one in SCT case) and the total width of the weighting function for SCT and MCDS are respectively:

$$R_{SCT} = \frac{FT_{SCT}}{W_{SCT}} = \frac{(\alpha - 1)}{(\alpha + 1)}$$

and

$$R_{MCDS} = \frac{FT_{MCDS, Eff}}{W_{MCDS, Eff}} = \frac{(\alpha - 1)(1 - \delta)}{(\alpha + 1) + (\alpha - 1)\delta}$$

Fig. 12 shows  $R_{MCDS}$  as function of  $k$ .

Now we can consider the  $T_{CYCLE}$  of the two systems proportional to the nominal length of the weighting functions:

$$T_{CYCLE, SCT} = n_{SCT} W_{SCT}$$

and

$$T_{CYCLE, MCDS} = n_{MCDS} W_{MCDS}$$

where  $n_{SCT} \geq 1$  and  $n_{MCDS} > 1$ .

This corresponds to the real case in which, in order to modify the width of the weighting function, we change the clock that controls the timing of the switches of the system. In this way, to change the shaping time of the filter, we change the  $T_{CYCLE}$  of the wheel.

As an ideal case  $n_{SCT}$  could be equal to 1, and the weighting functions of the same wheel could be one just next to the other. Actually, as already mentioned, we need some time to acquire the output signal and to reset the different stages of the circuit. With regard to  $n_{MCDS}$  it can't be 1 even in the ideal case, since, from the instant the weighting function reaches its nominal width, we have to wait at least  $(\alpha - 1)\tau\delta$  before the beginning of the next weighting function.

So in the ideal case it is:

$$n_{MCDS} = \frac{(\alpha + 1) + (\alpha - 1)\delta}{(\alpha + 1)}$$

Under these assumptions it follows that the number of wheels  $N_{SCT}$  and  $N_{MCDS}$  needed by the two systems to cover the time axis are respectively:

$$N_{SCT} = \frac{n_{SCT} W_{SCT}}{Flat - Top_{SCT}} = n_{SCT} \frac{(\alpha + 1)}{(\alpha - 1)}$$

and

$$N_{MCDS} = \frac{n_{MCDS} W_{MCDS}}{Flat - Top_{Eff, MCDS}} = n_{MCDS} \frac{(\alpha + 1)}{(\alpha - 1)(1 - \delta)}$$

So we have:

$$N_{MCDS} = \frac{n_{MCDS}}{n_{SCT}} \frac{1}{(1 - \delta)} N_{SCT}$$

In the ideal case:

$$N_{MCDS} = \frac{(\alpha + 1) + (\alpha - 1)\delta}{(\alpha + 1)(1 - \delta)} N_{SCT} \quad (3)$$

Obviously, if  $\varepsilon$  and  $k$  are such that  $\delta = 0$ , it is  $N_{MCDS} = N_{SCT}$ . It can be noticed that  $N_{MCDS}/N_{SCT}$  is equal to the ratio  $R_{SCT}/R_{MCDS}$ .

It is now convenient to plot the curves of the ENC of the two systems as functions of the total width  $W_T$  of the weighting functions. It will be in this way possible to compare the two implementations when they are working with the same counting rate.

The detector-preamplifier noise parameters used to plot the curves are

$$\begin{aligned} a &= 1.28 \cdot 10^{-27} \text{ V}^2/\text{Hz} \\ b &= 8 \cdot 10^{-31} \text{ A}^2/\text{Hz} \\ a_f &= 5 \cdot 10^{-12} \text{ V}^2 \\ b_f &= 0 \\ C_i &= 270 \text{ fF} \end{aligned}$$

All the values of  $\delta$  refer to  $\varepsilon = 1\%$ .

Fig. 13a shows the MCDS and SCT curves for  $\alpha = 2$  and  $k = 0.076$ . In this case  $\delta = 0$  and we have the same number of wheels in the two systems. However  $ENC_{MCDS}$  is bigger than  $ENC_{SCT}$  for every value of

$W_T$ . In particular at  $W_T=1.6\mu\text{s}$  (a reasonable value for ROTOR applications) it is:

$$\text{ENC}_{\text{SCT}}=17 \text{ e-} \quad \text{and} \quad \text{ENC}_{\text{MCDS}}=20 \text{ e-}$$

At this point, to reduce  $\text{ENC}_{\text{MCDS}}$  we must reduce the bandwidth of the circuit. On the other hand, increasing  $k$ ,  $R_{\text{MCDS}}$  decreases ( $\delta$  increases) at the same time. For example (Fig13b) with  $k=0.2$  and  $\alpha=2$  at  $1.6\mu\text{s}$   $\text{ENC}_{\text{SCT}}=\text{ENC}_{\text{MCDS}}=17 \text{ e-}$ . In these conditions  $\delta=0.447$  and  $N_{\text{SCT}}=2N_{\text{MCDS}}$ . So if we use 4 wheels to cover the time axis with SCT, we should use 8 wheels with MCDS.

As it is evident from (3) and table 2, increasing  $\alpha$ ,  $N_{\text{MCDS}}/N_{\text{SCT}}$  decreases. With  $k=0.2$  and  $\alpha=3$  (Fig. 13c),  $\text{ENC}_{\text{SCT}}$  is still very close to  $\text{ENC}_{\text{MCDS}}$  at  $1.6\mu\text{s}$ . Now  $\delta=0.23$  and  $N_{\text{MCDS}}=1.43N_{\text{SCT}}$ . So, again, if  $N_{\text{SCT}}=4$  we should use 6 wheels with MCDS. However, having increased the flat-top length, ENC has also increased for both the systems (19 e-). Fig. 13d shows the ENC curves for different values of  $k$  and  $\alpha=2$ .

## 8. Conclusions and Remarks

At high counting rates, i.e. with narrow weighting functions (e.g.  $W_T=1.6 \mu\text{s}$ ), with the same number of wheels and with small nominal flat-top slope (e.g.  $\varepsilon=1\%$ ) noise performances of SCT are better than MCDS.

On the other hand, with the same energy resolution (and the same counting rate) MCDS needs more wheels to cover the time axis than SCT.

Increasing  $\alpha$ , the difference in the number of wheels is reduced. However the ENC of both the systems increases.

The theoretical model was developed under the hypothesis to cover the time axis with the flat-tops of the wheels one next to the other. In practice a small superposition of flat-tops is necessary also in SCT. This can reduce the differences in the performances of the two systems.

Moreover there is an additional loss of resolution due to some small differences among the gains of the different wheels of the same chip. Measurements on first prototypes show that these differences are bigger in SCT Chips. This is probably due to the V-to-I

converter needed in SCT and to the tolerance of the integrated resistor needed to do the conversion.

It is also worthwhile pointing out that a MCDS system for synchronous signals has already been successfully implemented in CAMEX Amplifier used in ESA's XMM-Newton Mission. So, if SCT shows better theoretical noise performances (especially at high counting rates), MCDS finds, up to now, a simpler circuitual implementation.

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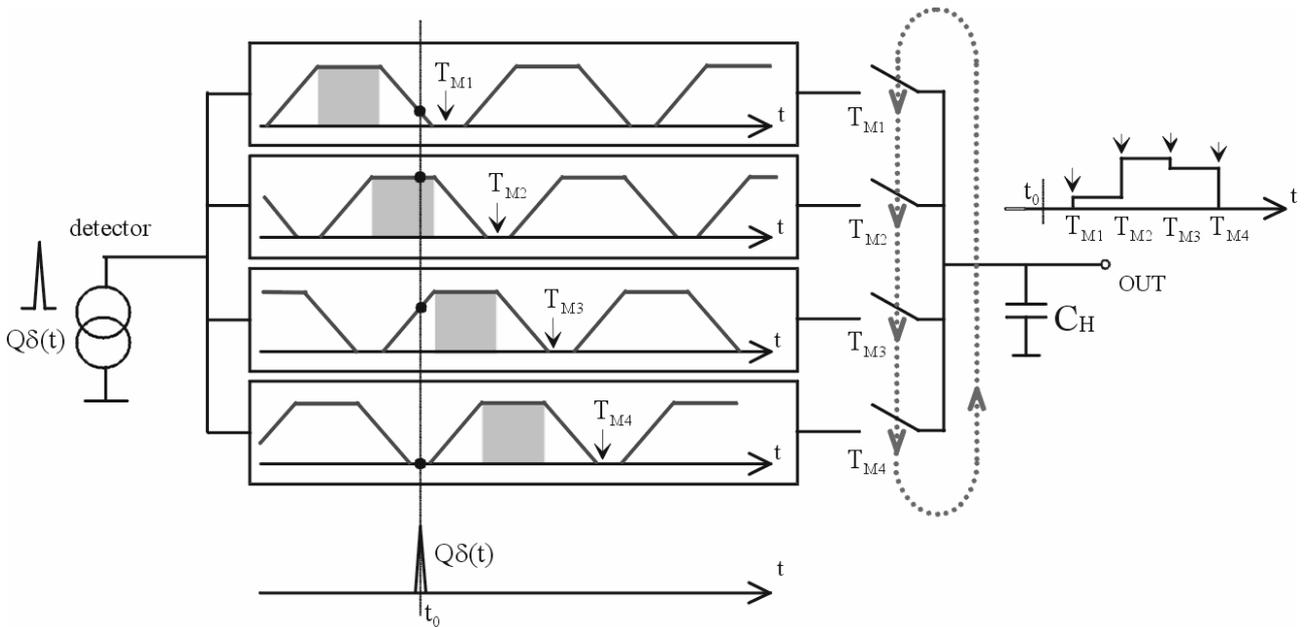


Fig. 1. Concurrent Wheel Technique working principle.

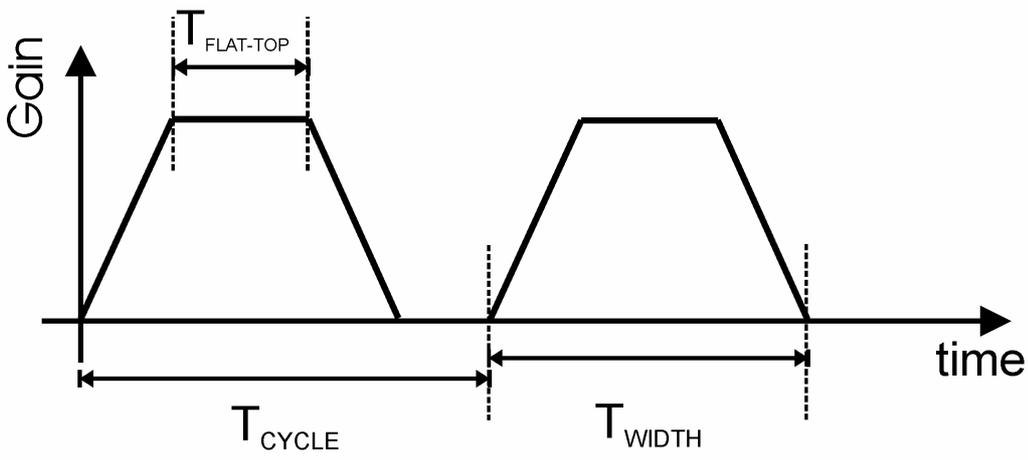


Fig. 2. CWT parameters for one wheel.

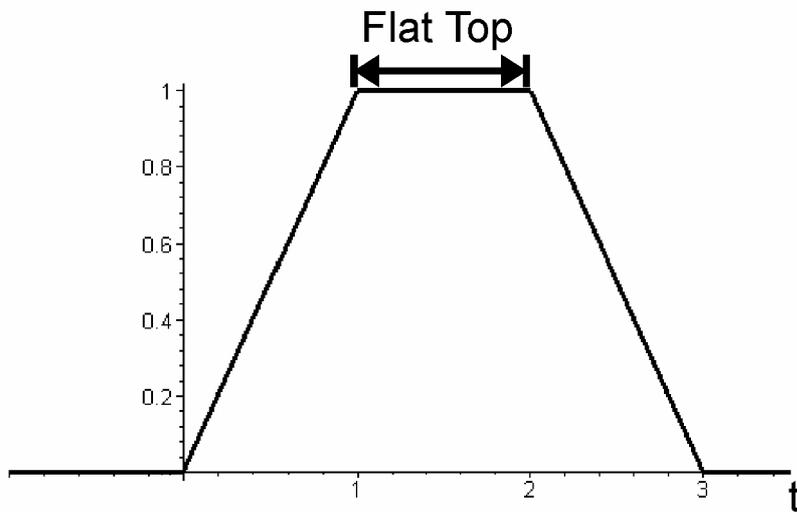


Fig. 3. SCT weighting function.

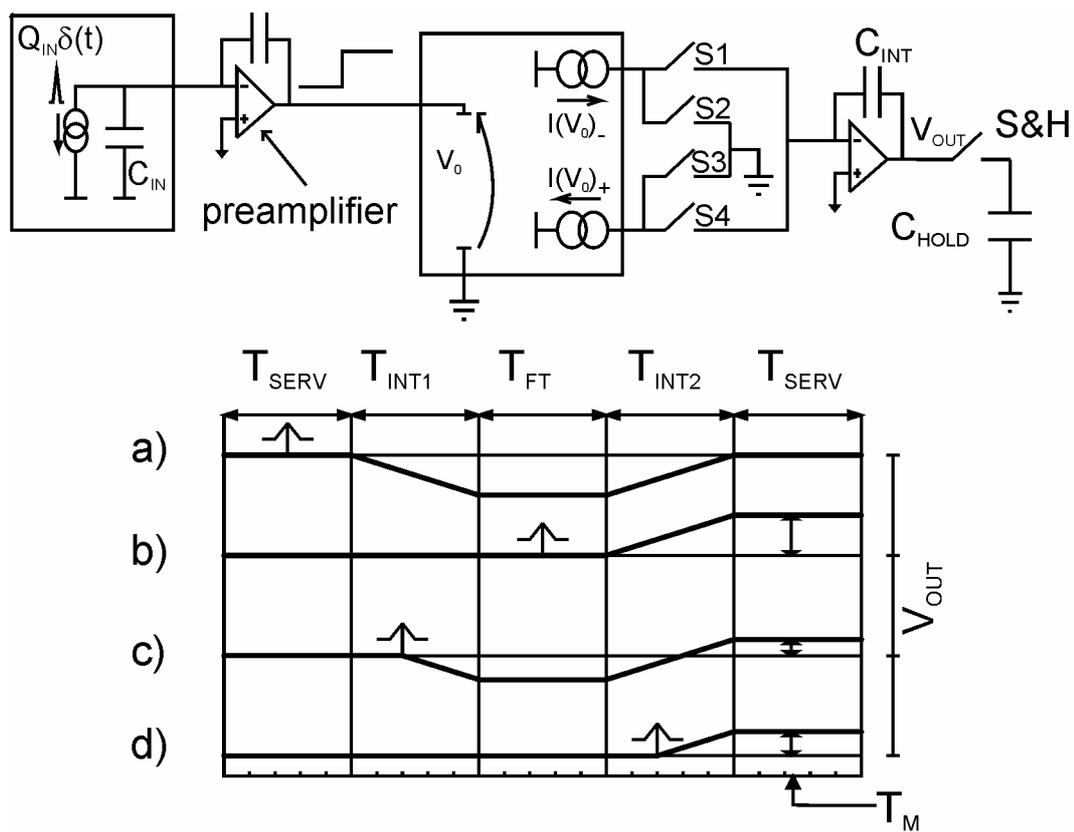


Fig. 4. SCT working principle. a) the input signal arrives before the first integration. b) the input signal arrives between the two integrations c,d) the signal arrives during one integration.

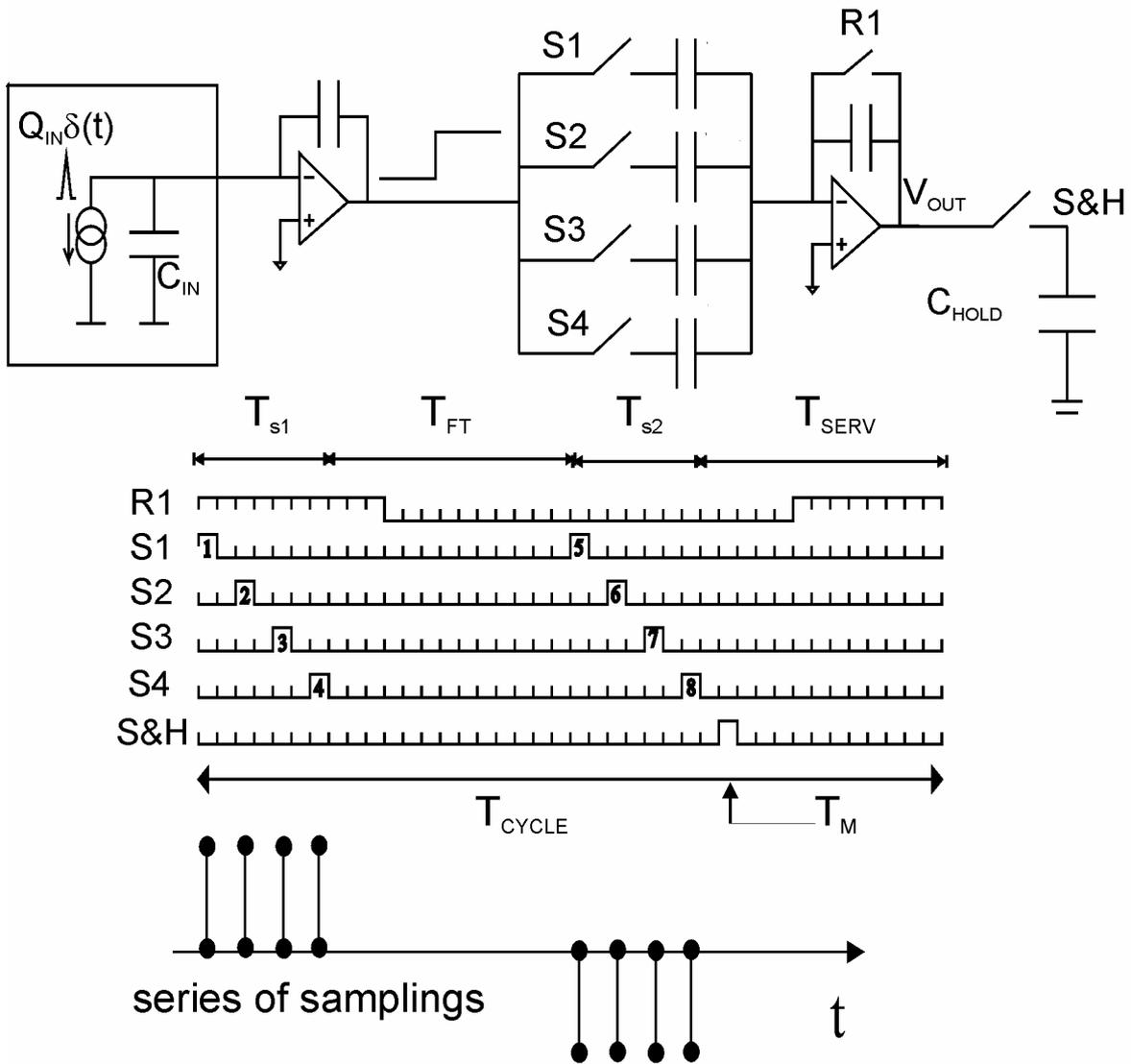


Fig. 5. MCDS working principle.

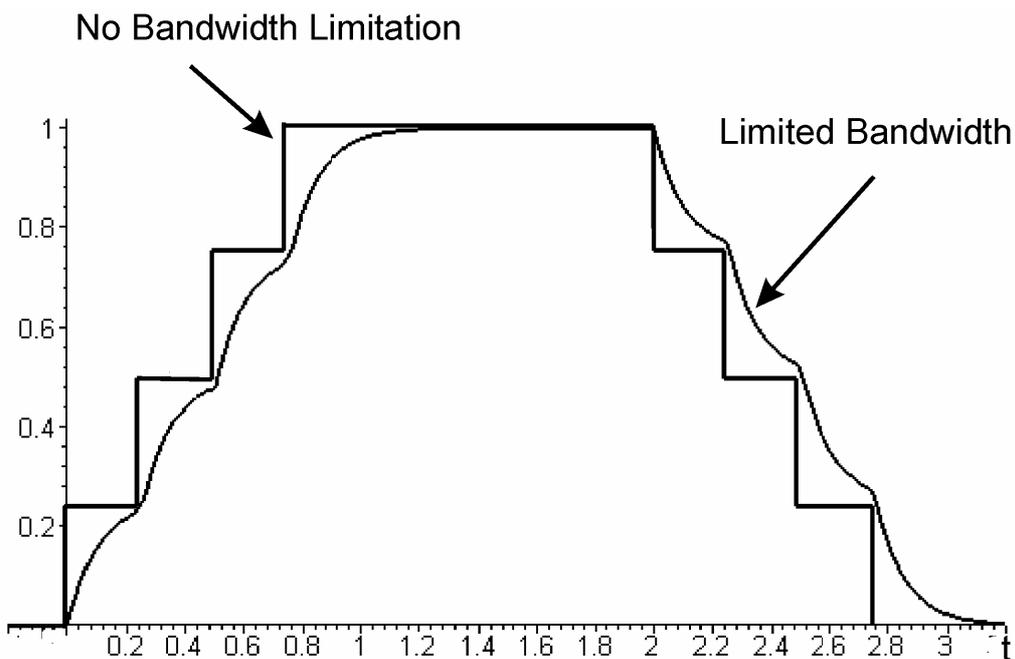
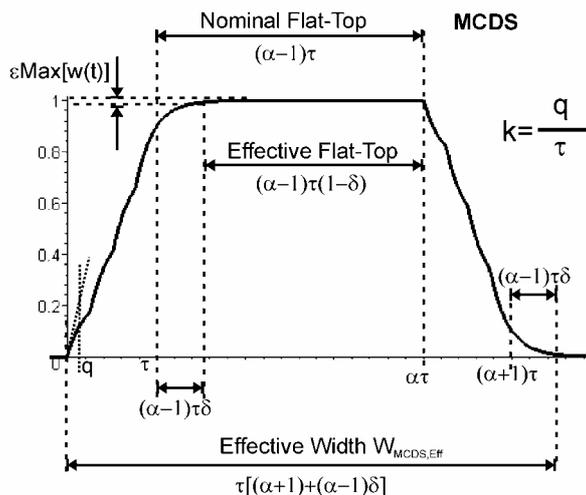
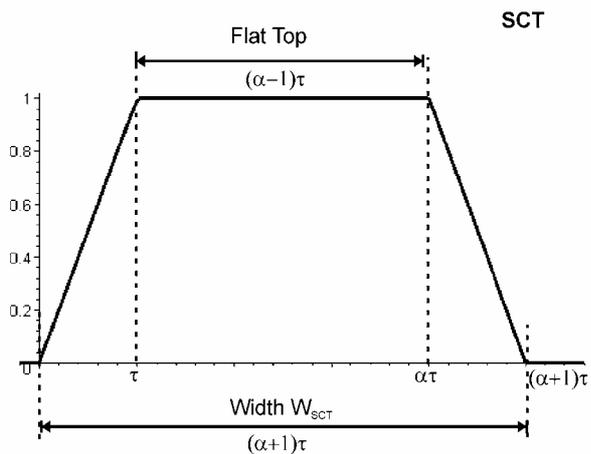


Fig. 6. MCDS folded weighting functions.



a) SCT

b) MCDS

Fig. 7. Definitions used in the calculations.

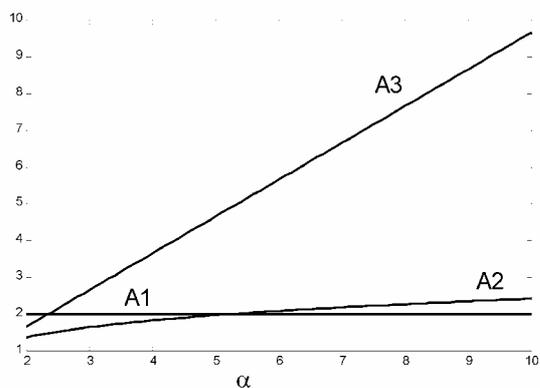


Fig. 8. A1, A2 and A3 for SCT as functions of  $\alpha$ .

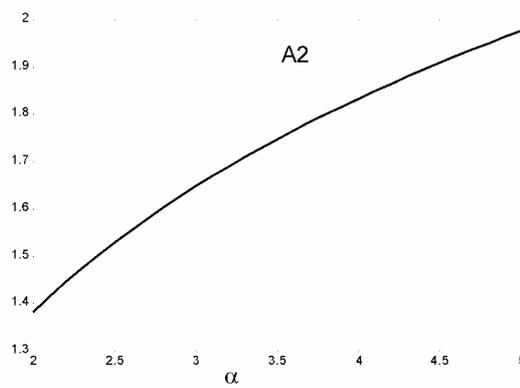


Fig. 9. A2 for SCT.

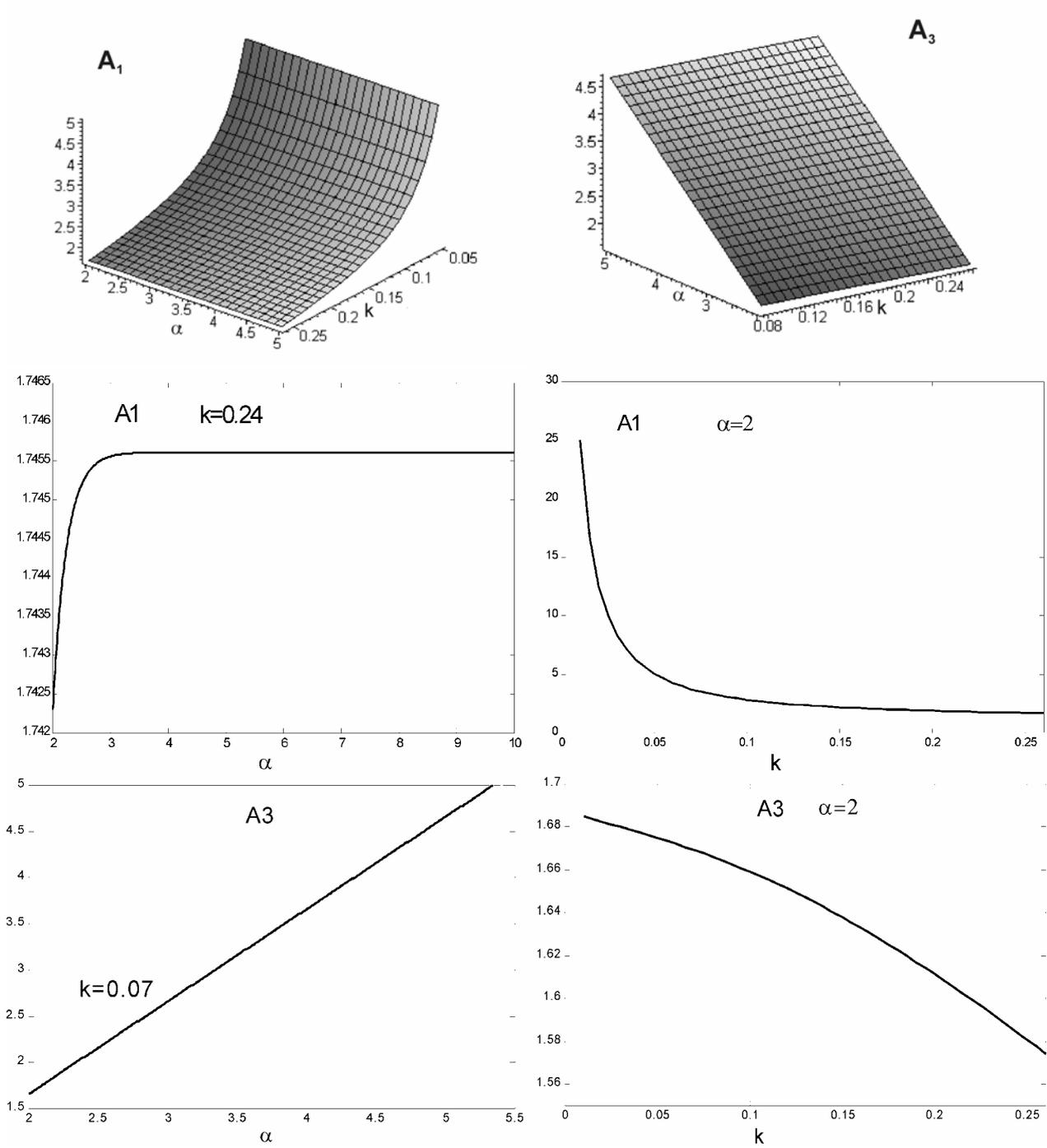


Fig. 10.  $A_1$  and  $A_3$  as functions of  $\alpha$  and  $k$  for MCDS

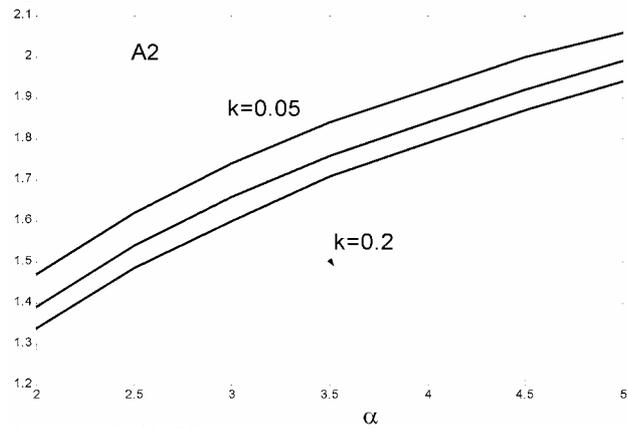


Fig. 11.  $A_2$  for MCDS.

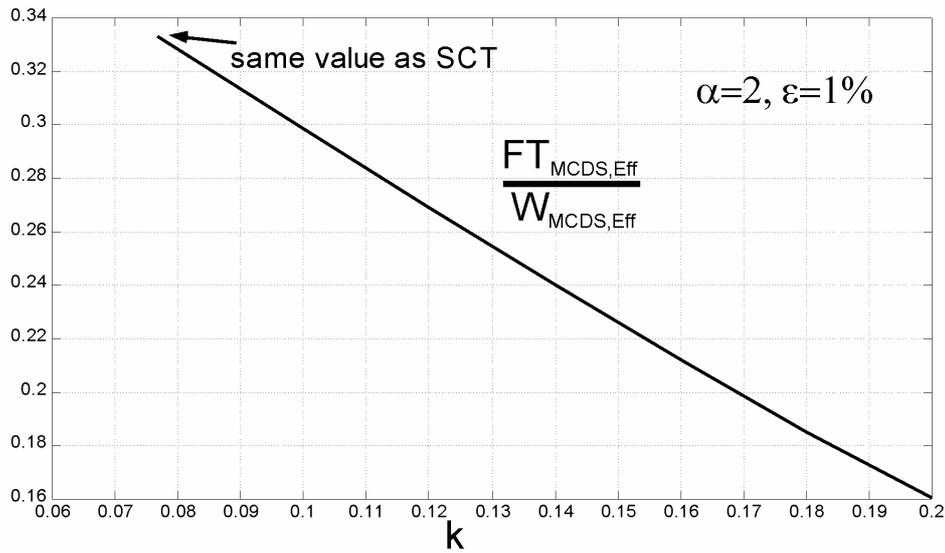


Fig. 12. Ratio between the nominal flat-top and the nominal length of MCDS weighting function.

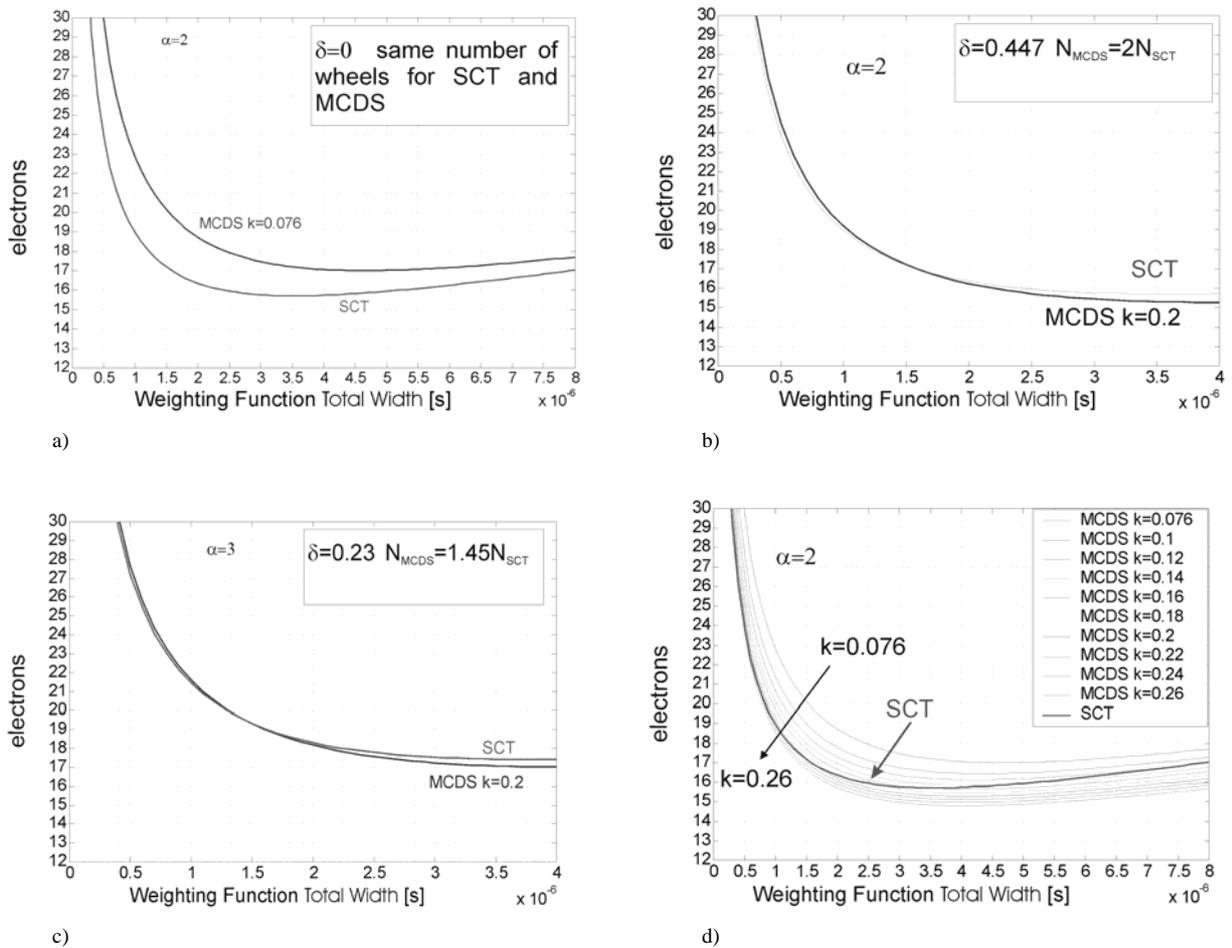


Fig. 13. ENC curves for SCT and MCDS with different values of  $\alpha$  and  $k$ .  $\delta$  refers to  $\epsilon=1\%$ .

$\alpha$	k	A1	A2	A3
2	0.05	5.05	1.47	1.67
	0.10	2.83	1.39	1.66
	0.15	2.20	1.34	1.64
	0.20	1.90	1.29	1.61
3	0.05	5.05	1.74	2.67
	0.10	2.83	1.66	2.66
	0.15	2.20	1.60	2.64
	0.20	1.90	1.56	2.61
4	0.05	5.05	1.92	3.67
	0.10	2.83	1.84	3.66
	0.15	2.20	1.79	3.63
	0.20	1.90	1.75	3.61
5	0.05	5.05	2.06	4.67
	0.10	2.83	1.99	4.66
	0.15	2.20	1.94	4.64
	0.20	1.90	1.89	4.61

Table 1. Noise coefficients for MCDS rounded off to the second decimal place. A2 has been calculated numerically. Note that for the chosen values of k, the coefficient A1 doesn't change within the considered  $\alpha$  range, at least up to the second decimal place.

$\alpha \backslash k$	0.076	0.1	0.12	0.14	0.16	0.18	0.2	0.22	0.24
2	0	0.080	0.152	0.225	0.300	0.375	0.447	0.515	0.578
3	0	0.040	0.076	0.113	0.151	0.190	0.230	0.279	0.311
5	0	0.020	0.038	0.057	0.076	0.095	0.115	0.135	0.156

$\delta(\alpha, k)$  for  $\varepsilon=1\%$

Table 2.  $\delta$  as function of  $\alpha$  and k with  $\varepsilon=1\%$ .